1 Introduction

This report describes experiments with the 2D Haar Wavelet, reduced 2D Haar Wavelet and inverse 2D Haar Wavelet transformations, performed on three images. Additionally, transformations were executed twice consecutively to see the effect of repeated Haar Wavelet transformation on an input image.

2 Experiments

Using current implementation, when executing Haar Wavelet transformation more than once on an input image, no wavelet coefficients are detected in the transformation matrix. This leads to an empty output image when performing inverse 2D Haar Wavelet transformation.

Below are the results of experiments performed only single 2D Haar Wavelet transformation on three different images. Each experiment shows an original image, resulting Haar transformation matrix and the resulting output image when performing inverse Haar transformation on the Haar transformation matrix. For every experiment, second row of the same image shows results when Haar transformation matrix is rounded up to nearest integer (quantisation).

Image 1.
3 Discussion

From presented results it can be seen that reconstruction from the Haar transformation matrix is as good as the original image. In case if the Haar transformation matrix is reduced (quantised), the reconstruction quality degrades. Other types of quantisation could be considered (e.g. threshold instead of rounding of coefficients) for better results.

In the case of compression, using wavelet domain, the image can be compressed significantly. For example, table below shows zero elements of output matrices. The more zeros present in the image, the less space it occupies.

<table>
<thead>
<tr>
<th>Original</th>
<th>Haar Transformation Matrix</th>
<th>Reduced Haar Transformation Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image 1</td>
<td>0</td>
<td>2772</td>
</tr>
<tr>
<td>Image 2</td>
<td>0</td>
<td>2092</td>
</tr>
<tr>
<td>Image 3</td>
<td>150</td>
<td>1717</td>
</tr>
</tbody>
</table>

Amount of zero elements in image based on the base (original image, Haar Transformation Matrix, Reduced Haar Transformation Matrix)

As it can be seen from the table, compression can be achieved when transforming an image in wavelet domain, from which image may be reconstructed without losing any information. The size may be reduced even more by by rounding the coefficients or by removing high frequencies (e.g. nullify coefficients that are over some threshold) which will lead to lossy compression.
I = im2double(imread('image.extension'));
I = imresize(I,[256 256]);

%disp('original image zeros');
%sum(I(:)==0)

[rows, cols] = size(I);

A = ConstructHaarWaveletTransformationMatrix(rows);
G = A';
%isequal((A.*G),eye(rows))
iterations = 1;

for t=1 : iterations;
    B=zeros(rows, cols);C=zeros(rows, cols);
    for r=1 : rows;
        if t == 1
            row=I(r,:);
        else
            row=C(r,:);
        end
        B(r,:)=A*row';
    end

    for r2=1 : cols;
        column=B(:,r2);
        C(:,r2)=A*column;
    end
end

%disp('wavelet domain zeros');
%sum(C(:)==0)

C=round(C);
%disp('reduced wavelet domain zeros');
%sum(C(:)==0)

%Inverse Haar transform
for k=1 : iterations;

E = zeros(rows, cols); F = zeros(rows, cols);
for r2 = 1 : rows;
    if k == 1
        column = C(r2, :);
    else
        column = E(r2, :);
    end
    F(r2, :) = G * column';
end
for r = 1 : rows;
    row = F(:, r);
    E(:, r) = G * row;
end

fprintf('reconstructed zeros');
sum(E(:) == 0)

subplot(1, 3, 1);
imshow(I);
title('Original Image');
subplot(1, 3, 2);
imshow(C);
title('Reduced Haar Transform');
subplot(1, 3, 3);
imshow(E);
title('Inverse Haar Transform');

function HaarTransformationMatrix = ConstructHaarWaveletTransformationMatrix(WidthOfSquareMatrix)
    Level = log2(WidthOfSquareMatrix);
    H = [1];
    NC = 1/sqrt(2); % normalization constant
    LP = [1 1];
    HP = [1 -1];
    for i = 1:Level
        H = NC * [kron(H, LP); kron(eye(size(H)), HP)];
    end
    HaarTransformationMatrix = H;